

SIR GILBERT WALKER AND A CONNECTION BETWEEN EL NIÑO AND STATISTICS

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Reference: Katz, R.W. (2002). *Statistical Science*, V. 17, pp. 97-112.

OPENING QUOTE

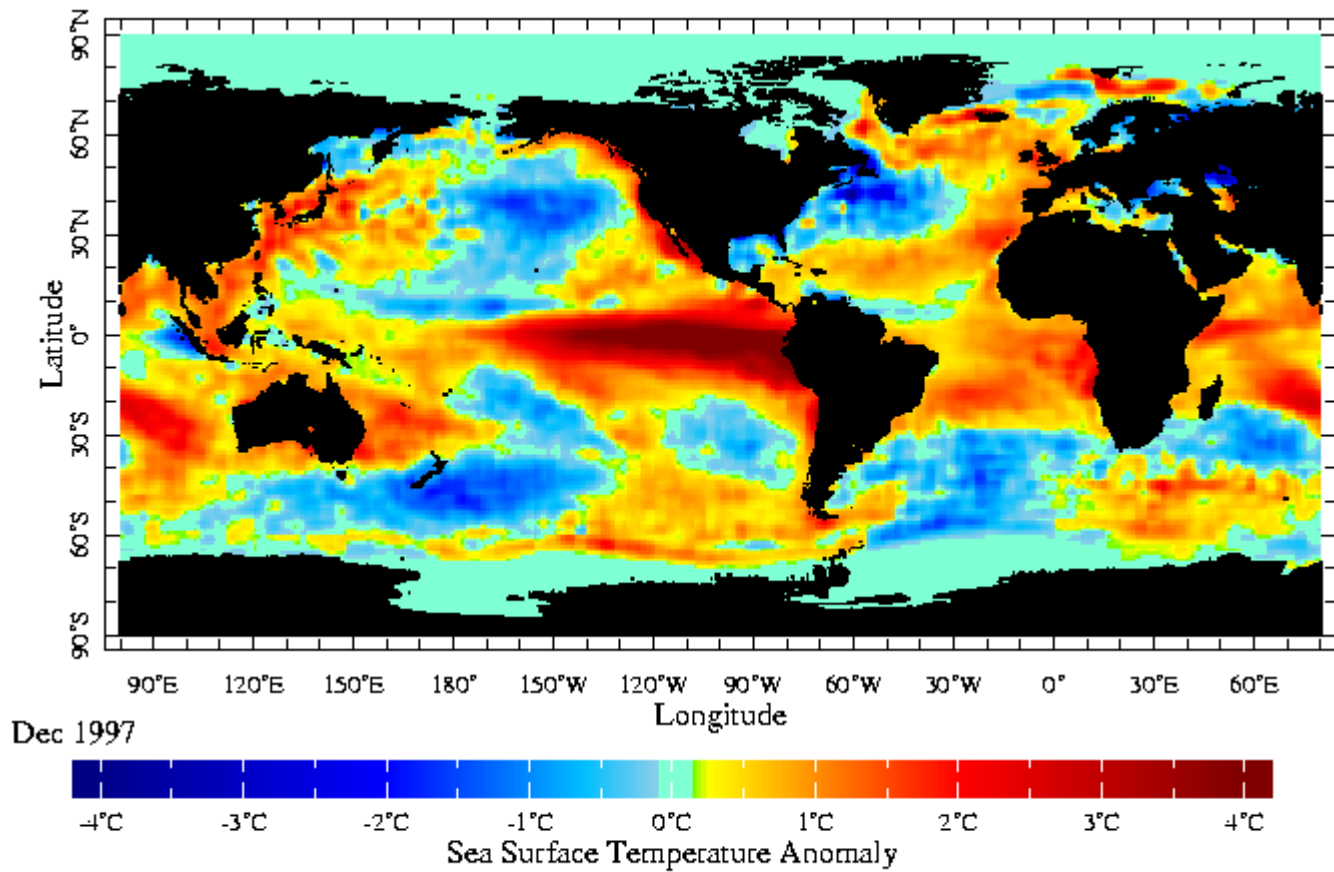
- Sir Gilbert Walker (1925)

-- *“It is a natural supposition that there should be in weather free oscillations with fixed natural periods, and that these oscillations should persist except when some external disturbance produces discontinuous changes in phase or amplitude.”*

OUTLINE

- (1) Background**
- (2) World Weather / Centers of Action**
- (3) Yule-Walker Equations**
- (4) Reaction to Walker's Work**
- (5) Present Situation**





(1) BACKGROUND

- **Quote About Cell Phones (Thierry Dutoit, 2004)**
 - ***“Every cell phone call solves the Yule-Walker equations every ten microseconds”***
- **Quote about ENSO phenomenon (Jacob Bjerknes, 1969)**
 - ***“The Walker Circulation . . . must be part of the mechanism of the still larger Southern Oscillation”***

- **El Niño - Southern Oscillation (ENSO) Phenomenon**

- **El Niño (“Christ Child”)**

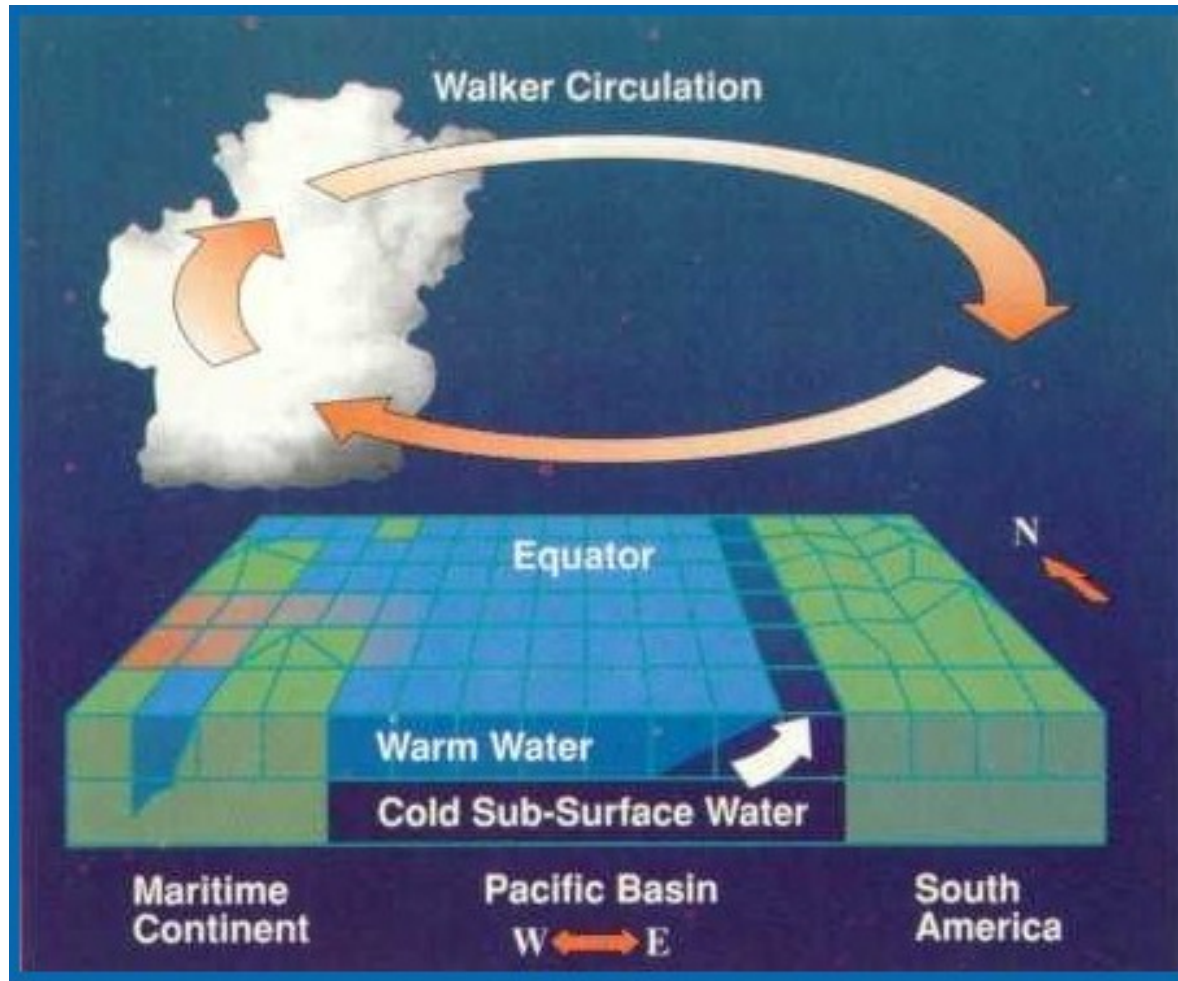
- **Teleconnections**

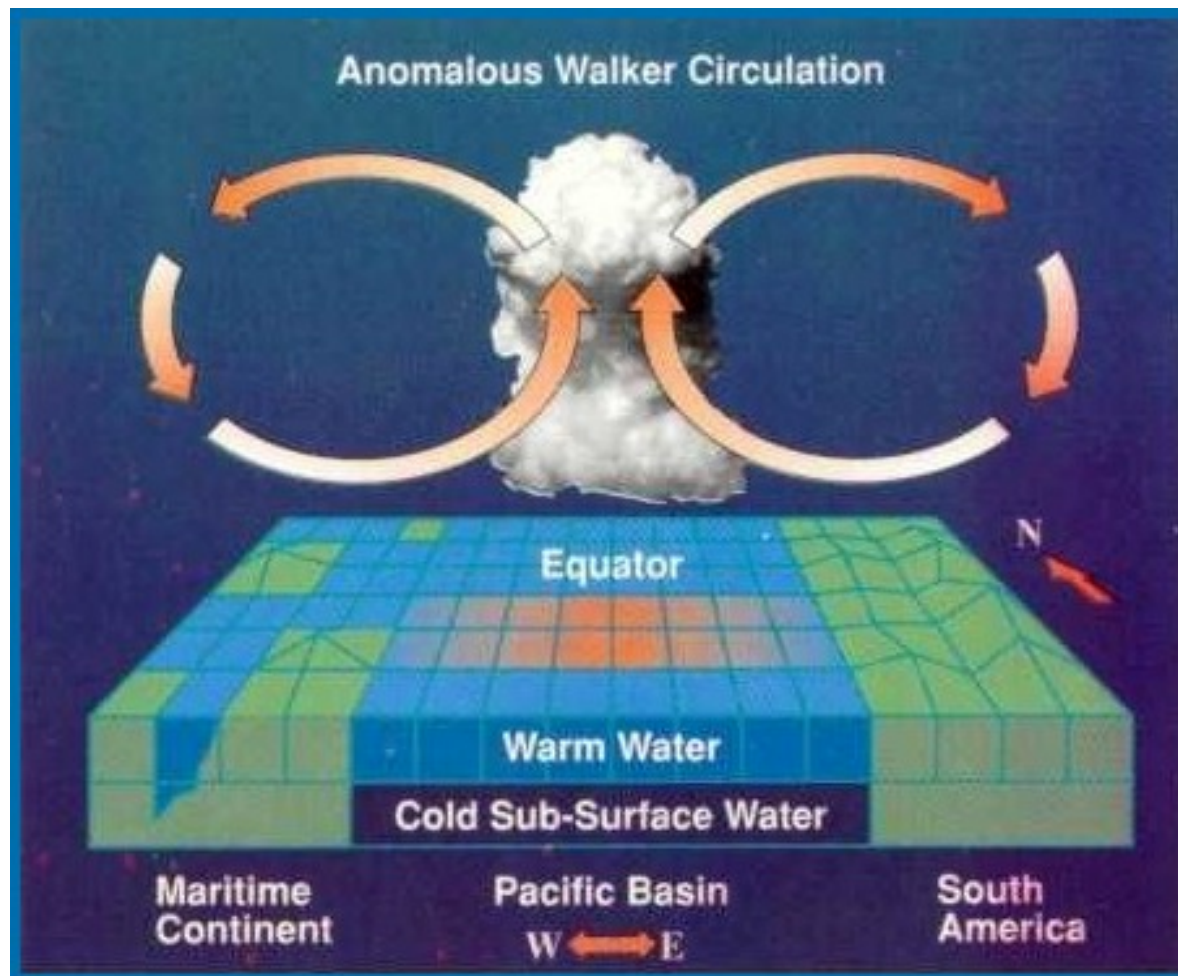
- **Centers of action / Pressure seesaw**

- Southern Oscillation (SO)**

- (Named by Walker)**

- Walker Circulation (Bjerknes, 1969)**





- **Quasiperiodic Behavior**

-- p^{th} - order autoregressive process [AR(p)]

-- “Yule-Walker Equations” (Kendall, 1949)

Yule (1927): **AR(2)** model for sunspots

Walker (1931): **AR(4)** model for Darwin pressure

Use today: **Partial autocorrelations**
(e. g., **R** statistical programming language)

Speech compression

(2) WORLD WEATHER / CENTERS OF ACTION

- **Training and Career of Sir Gilbert Walker**

- **1868** **Born (Lancashire, England)**
- **1886-1890** **Studied mathematics, Trinity College**
- **1892** **Published first paper**
- **1895-1903** **Lecturer, Trinity College**
- **1903-1924** **India Meteorological Department**
- **1924-1934** **Imperial College, University of London**
- **1934** **Retirement**
- **1950** **Published last paper**
- **1958** **Died (Surrey, England)**



- **Statistical Research of Walker**

- **Correlation and regression**

 - Southern Oscillation (SO)**

 - North Atlantic Oscillation (NAO)**

 - North Pacific Oscillation (NPO)**

- **Multiple comparisons**

 - Walker Test (Correlations & Periodogram)**

- **Harmonic analysis**

 - “Period-hunting” (Cautioned against)**

(3) YULE-WALKER EQUATIONS

- AR Process

-- $X_1, X_2, \dots, X_t, \dots$ zero mean $AR(p)$ process

-- $p = 2$ case

$$X_t = \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + a_t$$

φ_k k^{th} - order autoregression coefficient

a_t zero mean, uncorrelated (“white noise”) process

- Yule-Walker Equations ($p = 2$ case)

$$\rho_k = \varphi_1 \rho_{k-1} + \varphi_2 \rho_{k-2} \quad k \neq 0$$

$$\sigma_a^2 = (1 - \varphi_1 \rho_1 - \varphi_2 \rho_2) \sigma^2$$

$\rho_k = \text{corr}(X_t, X_{t-k})$ k^{th} - order autocorrelation coefficient

$\sigma^2 = \text{var}(X_t)$ Process variance

$\sigma_a^2 = \text{var}(a_t)$ Error (or “innovation”) variance

- **Quasiperiods**

- **AR(p) process, $p \geq 2$**

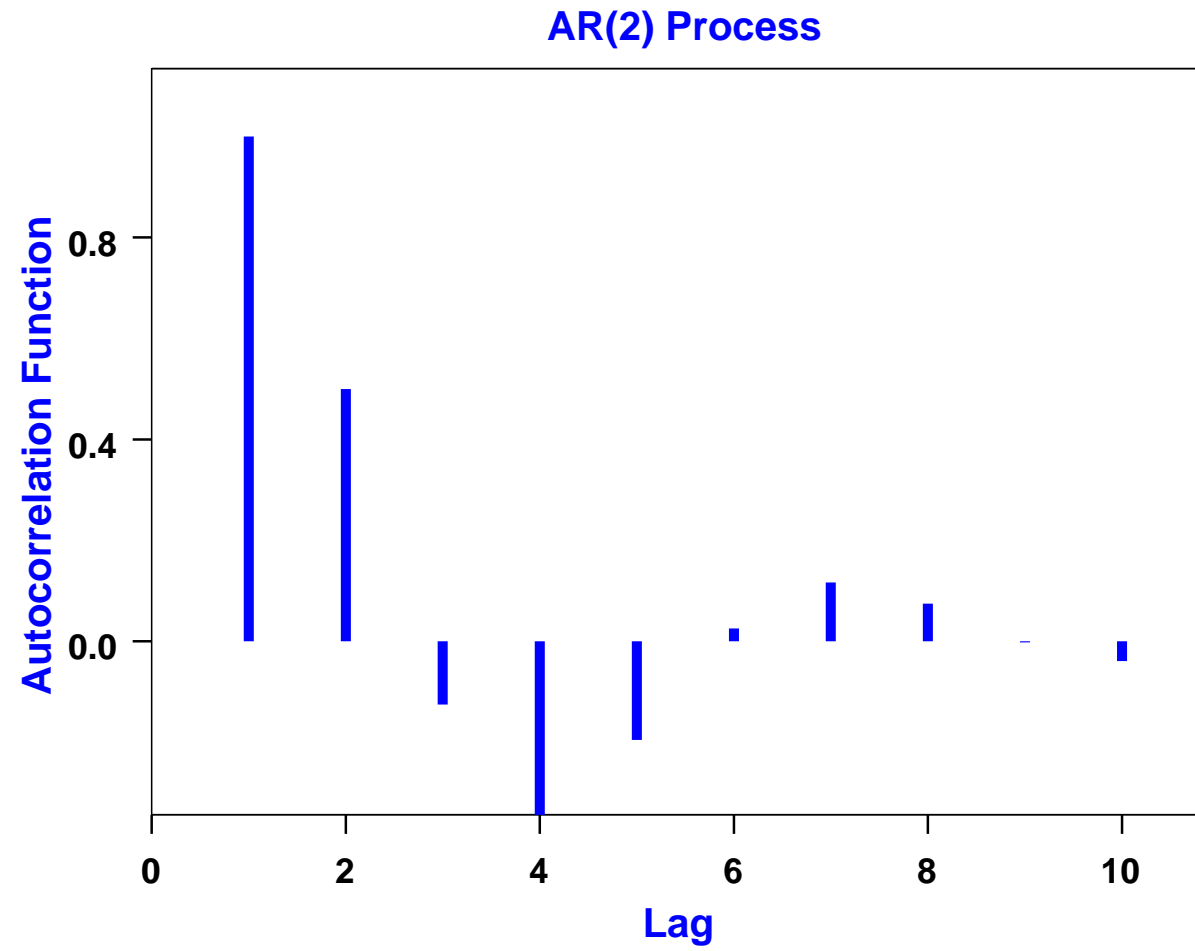
- Capable of producing quasiperiodic behavior**

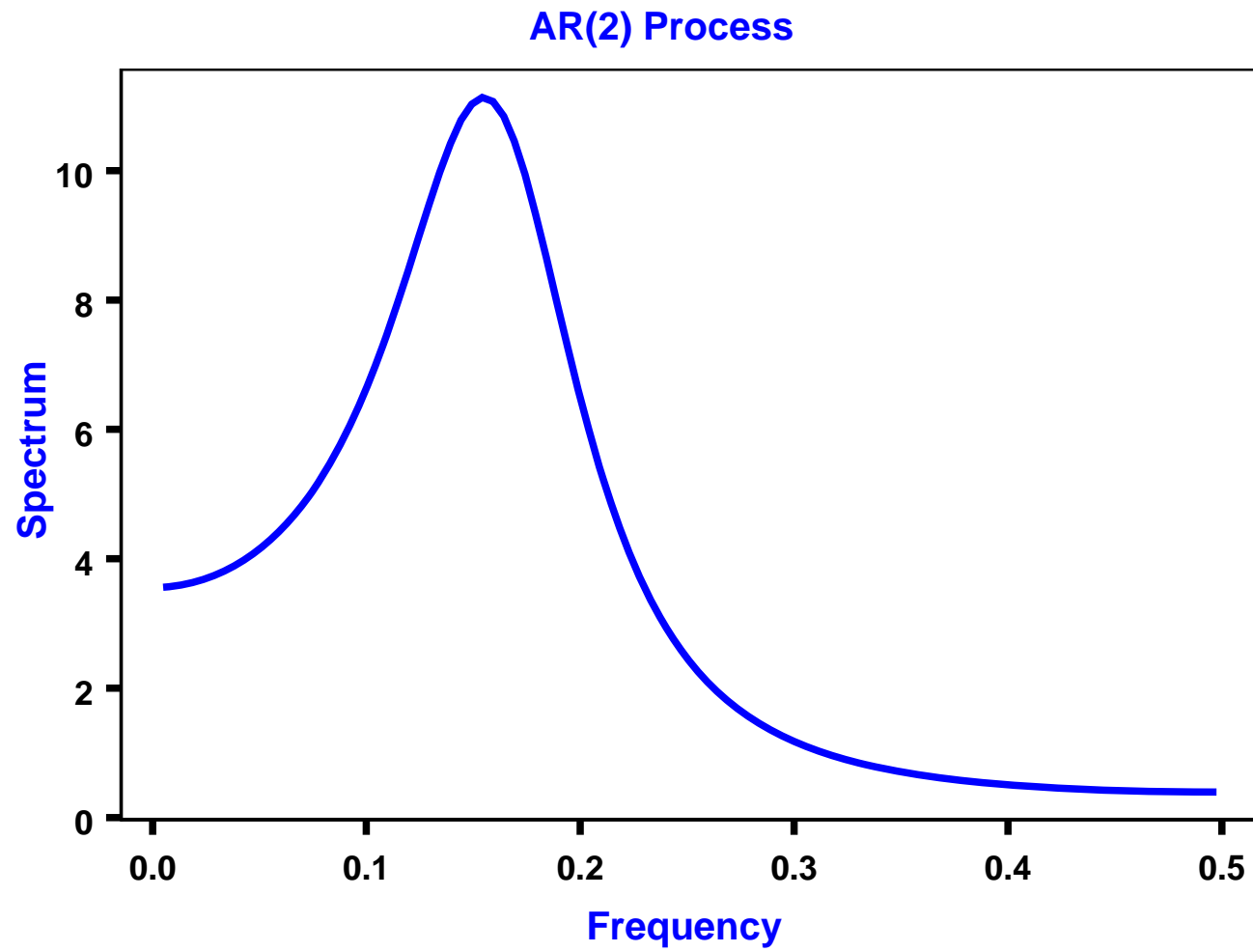
- **Correlation quasiperiod**

- Damped oscillation in autocorrelation function**

- **Spectral quasiperiod**

- Peak (Broad band) in spectrum**





- **Yule's Model for Sunspot Numbers**

- **Sine wave randomly shifted in phase and amplitude**

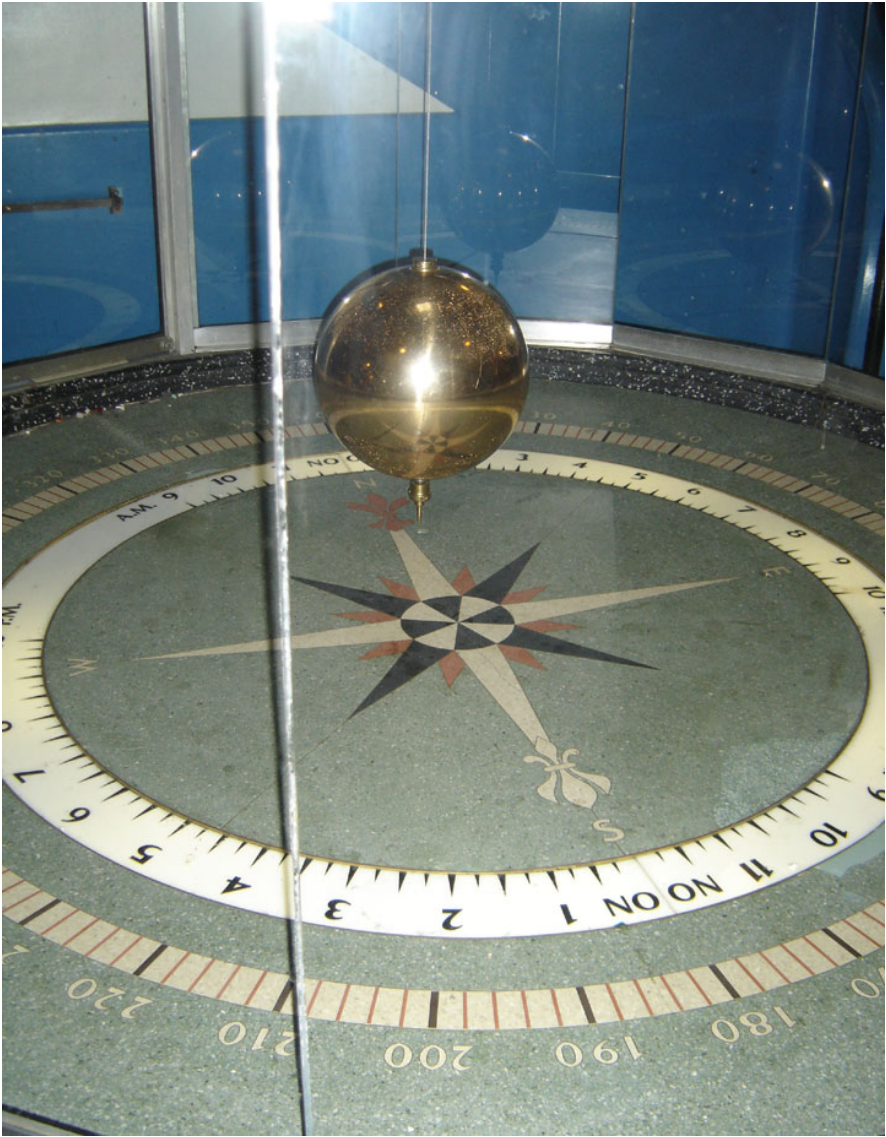
Analogy to pendulum:

“Unfortunately boys get into the room and start pelting the pendulum with peas, sometimes from one side and sometimes from the other”

- **AR(2) process**

Fit by least squares to annual sunspot time series, 1749 – 1924

(Correlation quasiperiod ≈ 11 yrs.)



- **Walker's Model for Darwin Pressure**

- **Seasonal index of Southern Oscillation**

- **Derivation of Yule-Walker Equations**

- Extended Yule's **AR(2)** model to general **AR(p)** process, $p \geq 1$

- **Analysis of Darwin pressure**

- Decided on **AR(4)** model

- Correlation quasiperiod ≈ 3 yrs.

- **Reanalysis of Darwin Pressure**

- **Modifications**

- Correct for instrumental bias**

- Remove annual cycle in standard deviation**

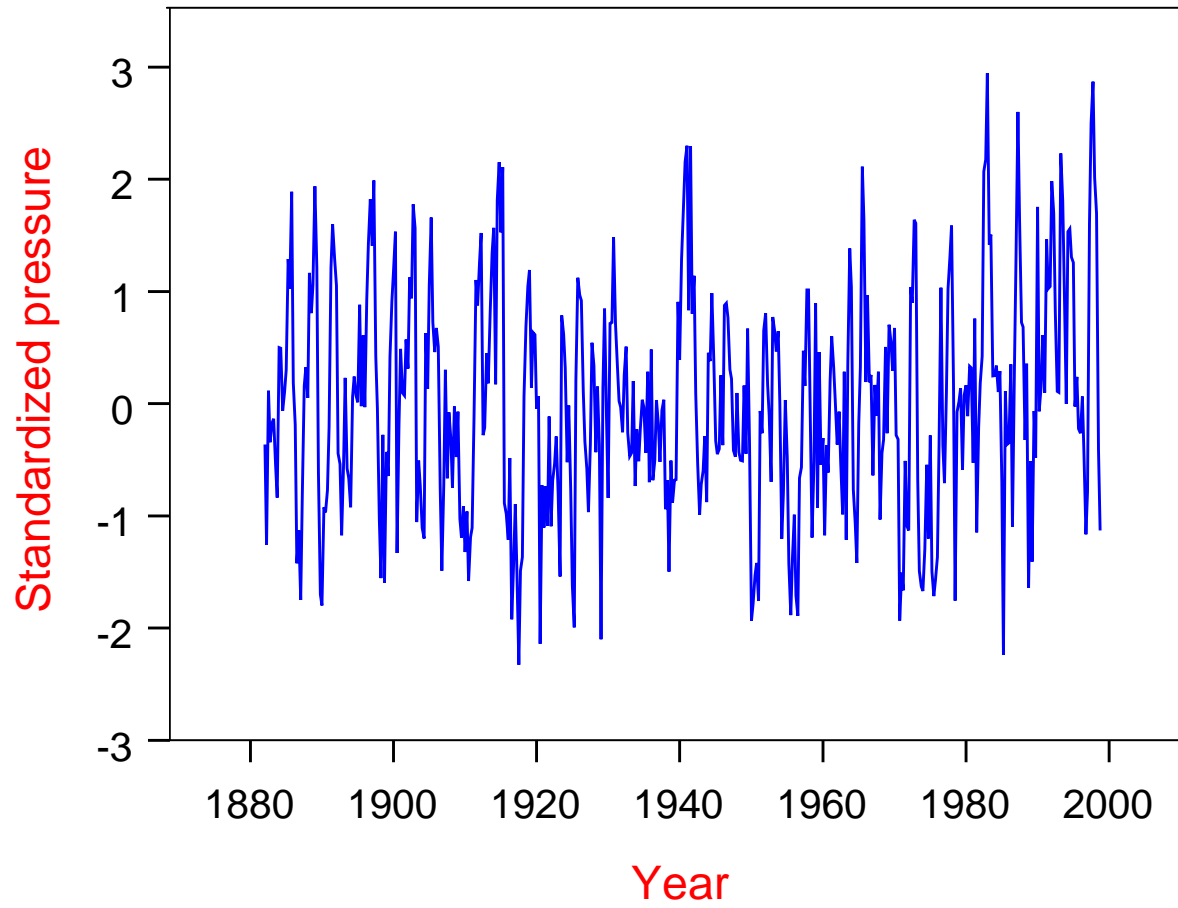
- Parameter estimation by Yule-Walker Equations**

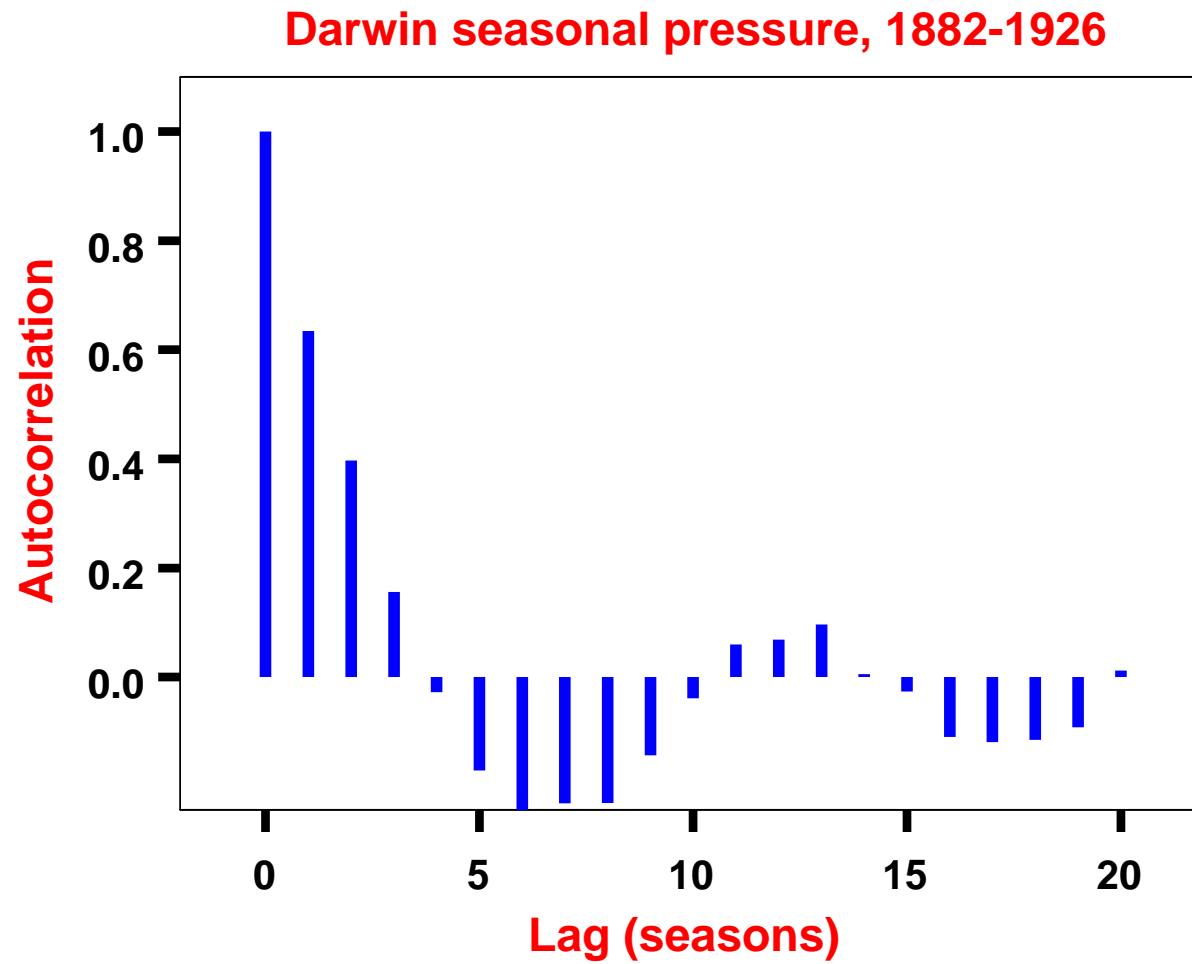
- **Results**

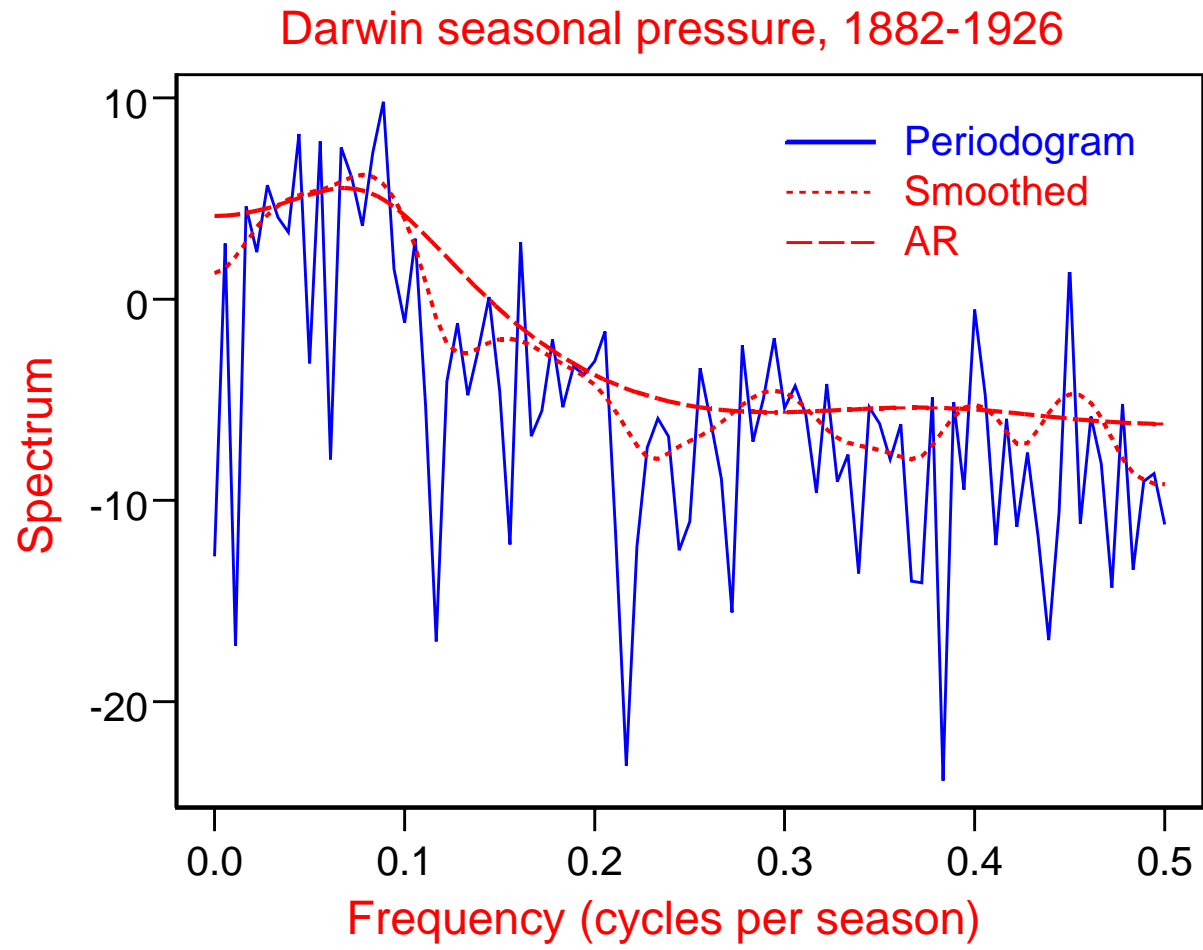
- Damped oscillation in autocorrelation function**

- AR(4) model gives spectral quasiperiod $\approx 3 - 3.5$ yrs**

Seasonal mean pressure at Darwin







(4) REACTION TO WALKER'S WORK

- **Reaction in Statistics**

-- Wold (1938) book marked emergence of modern time series analysis

Cited only two applications of **AR** processes:

(i) Yule (1927) sunspots application

AR(2) model praised by Wold

(ii) Walker (1931) pressure application

Wold thought **AR(1)** model adequate rather than Walker's **AR(4)**

(i. e., *no* quasiperiodic behavior)

- **Reaction in Meteorology**

- **General skepticism about statistics**

- **Physical explanation lacking (Walker sought, but appeals unheeded)**

- **Sheppard(1959)**

“Walker’s hope was presumably not only to unearth relations useful for forecasting but to discover sufficient and sufficiently important relations to provide a productive starting point for a theory of world weather. It hardly appears to be working out like that.”

(5) PRESENT SITUATION

- **Quasiperiodic Behavior**
 - **Consensus that ENSO phenomenon exhibits quasiperiodic behavior**
 - **Weakness of General Circulation Models**
(Unable to reproduce correct quasiperiod)
- **Nonlinear Dynamics**
 - **Quasiperiodic feature of ENSO viewed as fundamental**
 - **Source remains unclear**

CLOSING QUOTE

- **Sir Gilbert Walker (1927)**

-- ***“There is, to-day, always a risk that specialists in two subjects, using languages full of words that are unintelligible without study, will grow up not only, without knowledge of each other’s work, but also will ignore the problems which require mutual assistance.”***